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basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS/
NATIONAL SENIOR CERTIFICATE EXAMINATIONS

TECHNICAL MATHEMATICS P1

MAY/JUNE 2024

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages, a 2-page information sheet and 2 answer sheets.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of NINE questions.
2. Answer ALL the questions.
3. Answer QUESTION 4.1.3, QUESTION 4.1.5 and QUESTION 4.3 on the ANSWER SHEETS provided. Write your centre number and examination number in the spaces provided on the ANSWER SHEETS and hand in the ANSWER SHEETS with your ANSWER BOOK.
4. Number the answers correctly according to the numbering system used in this question paper.
5. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
6. Answers only will NOT necessarily be awarded full marks.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. If necessary, round off answers to TWO decimal places, unless stated otherwise.
9. Diagrams are NOT necessarily drawn to scale.
10. An information sheet with formulae is included at the end of the question paper.
11. Write neatly and legibly.

QUESTION 1

1.1 Given: $x^2 - x - 12 = p$

Solve for x if:

1.1.1 $p = 0$ (2)

1.1.2 $p \leq 0$ (1)

1.1.3 $p = -5$ (correct to TWO decimal places) (4)

1.2 Given: $2y - x = 7$ and $x^2 + xy = 21 - y^2$

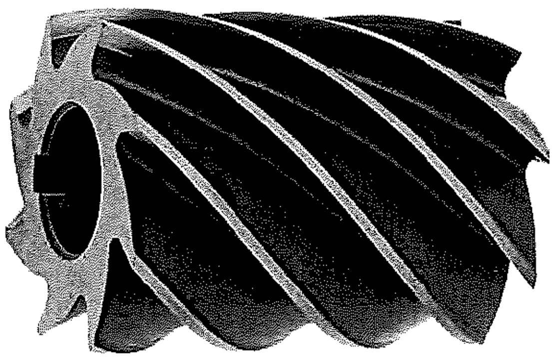
1.2.1 Make x the subject of the equation $2y - x = 7$ (1)

1.2.2 Hence, or otherwise solve for x and y. (5)

1.3 The picture below shows a milling cutter.

The formula used to determine the relationship between the number of teeth (T), the diameter (D) of the cutter and the depth (d) of the cutter is given by:

$$T = \frac{12{,}5\,D}{D + 4d}$$



T = Number of teeth
D = Diameter of the cutter in cm
d = Depth of the cutter in cm

1.3.1 Make d the subject of the formula. (3)

1.3.2 Hence, or otherwise, calculate the depth of the cutter (d) if $T = 10$ and $D = 32$ cm (2)

1.4 Evaluate $2\left(111110_2 + 38\right)$. Leave your answer in decimal form. (2)

[20]

QUESTION 2

2.1 Given: $T = \dfrac{\sqrt{2-5b}}{3b}$

Determine the numerical value of b, for which T is:

2.1.1 Undefined (1)

2.1.2 Equal to zero (1)

2.2 Determine the value(s) of k for which the equation $kx^2 = 35 - 2x$ has real roots. (5)
[7]

QUESTION 3

3.1 Simplify the following **without the use of a calculator**:

3.1.1 $\sqrt[3]{8x^{27}}$ (1)

3.1.2 $9^{n+1} \times 4^{n} \times 6^{1-2n}$ (3)

3.1.3 $\sqrt{k}\left(2-\sqrt{k}\right)-\sqrt{4k}$ (3)

3.2 Given: $\dfrac{\log 72 - \log 2}{\log 6}$

3.2.1 Write the following as a single logarithm:

$\log 72 - \log 2$ (1)

3.2.2 Hence, simplify **without using a calculator**: $\dfrac{\log 72 - \log 2}{\log 6}$ (2)

3.3 Solve for x: $5^{x+2} - 5^{x} = 600$ (5)

3.4 Given the complex numbers: $r_1 = 2 + 3i$ and $r_2 = i$

3.4.1 Write down the conjugate of r_2. (1)

3.4.2 Hence, simplify $\frac{r_1}{r_2}$. Show ALL steps. (4)

3.5 Write down the numerical values of a and b if $a + bi = -i - 14$ (2)
[22]

QUESTION 4

4.1 Given: Functions f and h defined by $f(x) = -2(x-3)^2 + 18$ and $h(x) = 2x + c$

4.1.1 Write down the coordinates of the turning point of f. (2)

4.1.2 Determine the x-intercepts of f. (4)

4.1.3 Hence, sketch the graph of f on the ANSWER SHEET provided. Clearly show the intercepts with the axes and coordinates of the turning point of f. (3)

4.1.4 A $(5;t)$ is a point of intersection of f and h.

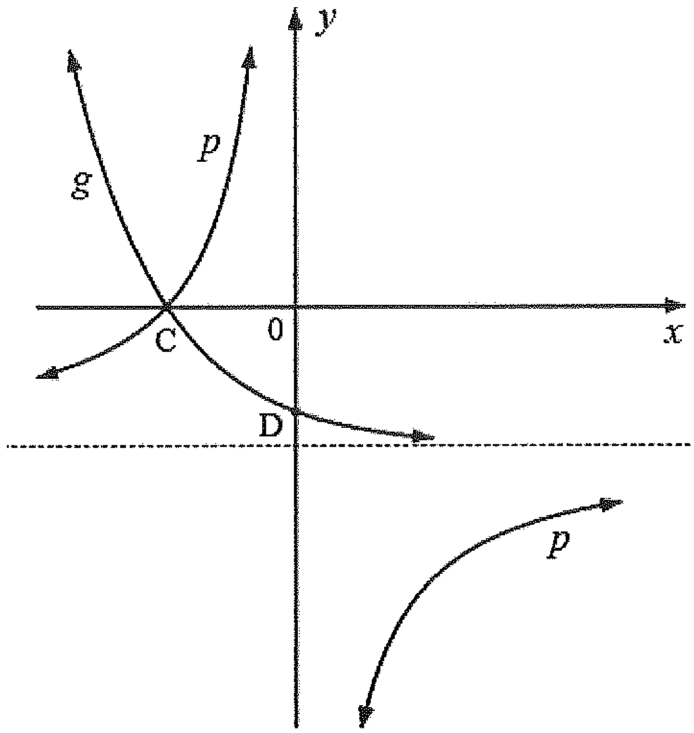
(a) Calculate the numerical value of t. (2)

(b) Hence, determine the numerical value of c. (2)

4.1.5 Sketch the graph of h on the same set of axes as graph f on the ANSWER SHEET provided. Clearly show the intercepts with the axes and the coordinates of A. (2)

4.2 The graph below represents functions p and g defined by $p(x) = -\frac{8}{x} - 4$ and $g(x) = a^x + q$

- Functions p and g have a common horizontal asymptote.
- D is the y-intercept of g and C the x-intercept of both p and g.



4.2.1 Write down:

(a) The domain of p (1)

(b) The range of g (1)

(c) The numerical value of q (1)

(d) The coordinates of D (2)

4.2.2 Determine the coordinates of C. (3)

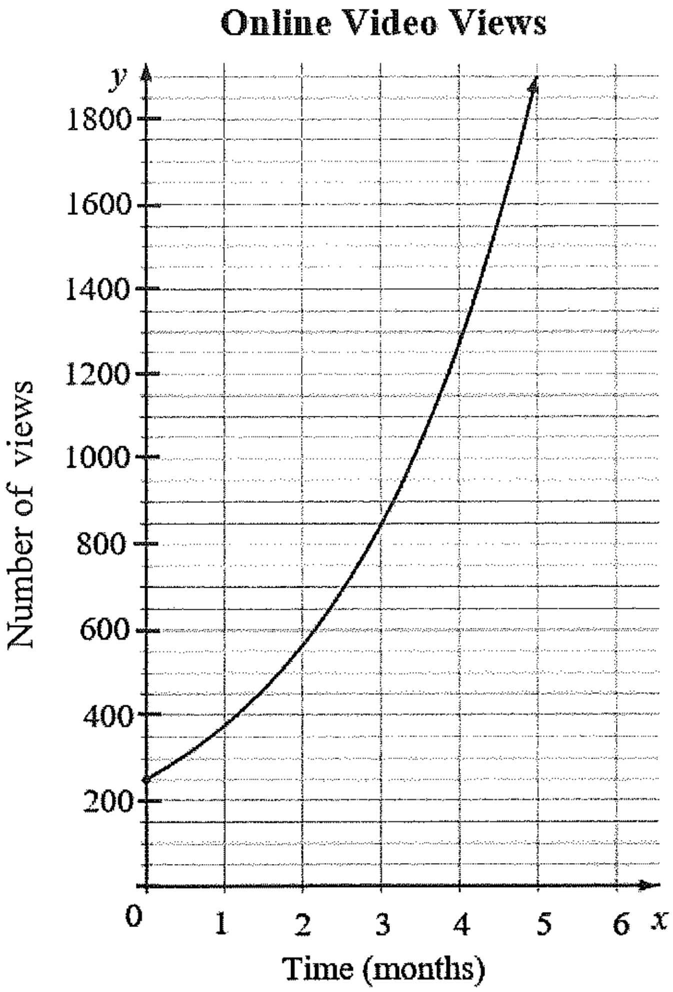
4.2.3 Determine the numerical value of a. (2)

4.3 The line $y = c$ is a tangent to semi-circle h defined by $h(x) = \sqrt{r^2 - x^2}$
The distance between the x-intercepts of h is 10 units.
Sketch the graph of h and the line $y = c$ on the ANSWER SHEET provided. Clearly show all intercepts with the axes. (3)

[28]

QUESTION 5

5.1 The exponential graph below shows the number of views of a popular online video, that grows at a compound rate of 50% per month. The number of months since the video was posted online is shown on the x-axis and the number of views on the y-axis.



5.1.1 Write down the number of views the online video received immediately after it was posted. (1)

5.1.2 Hence, calculate the total number of views the online video received at the end of the first year (rounded to nearest whole number). (3)

5.1.3 Online videos with **100 000 views** are considered to go viral.

Determine, to the nearest month, how long it will take (counting from the start) for this online video to go viral if the growth rate remains 50% per month. (4)

5.2 Timo wants to buy a welding machine in 5 years' time that will cost R23 000. He decides to invest R15 000 in an investment account.

- Interest rate for the first three years is 8,5% per annum, compounded quarterly.
- Interest rate for the next two years is 6% per annum, compounded semi-annually.

5.2.1 Determine whether the value of the investment at the end of five years will be enough to buy the welding machine. (5)

5.2.2 Hence, calculate the total interest earned by the investment over five years. (2)

[15]

QUESTION 6

6.1 Given: $f(x)=11+7x$

Determine $f^{\prime}(x)$ using FIRST PRINCIPLES. (5)

6.2 Determine:

6.2.1 $\dfrac{dy}{dx}$ if $y = x^8$ (1)

6.2.2 $f^{\prime}(x)$ if $f(x)=\sqrt[3]{x^4}$ (2)

6.2.3 $D_x\left[\dfrac{x^2-16}{4-x}\right]$ (4)

6.3 Determine the average gradient of the function defined by $g(x)=-\dfrac{9}{x}$ between $x=-3$ and $x=-1$ (3)

6.4 Given: $f(x)=mx^3+mx-4$

6.4.1 Determine $f^{\prime}(x)$ in terms of m (2)

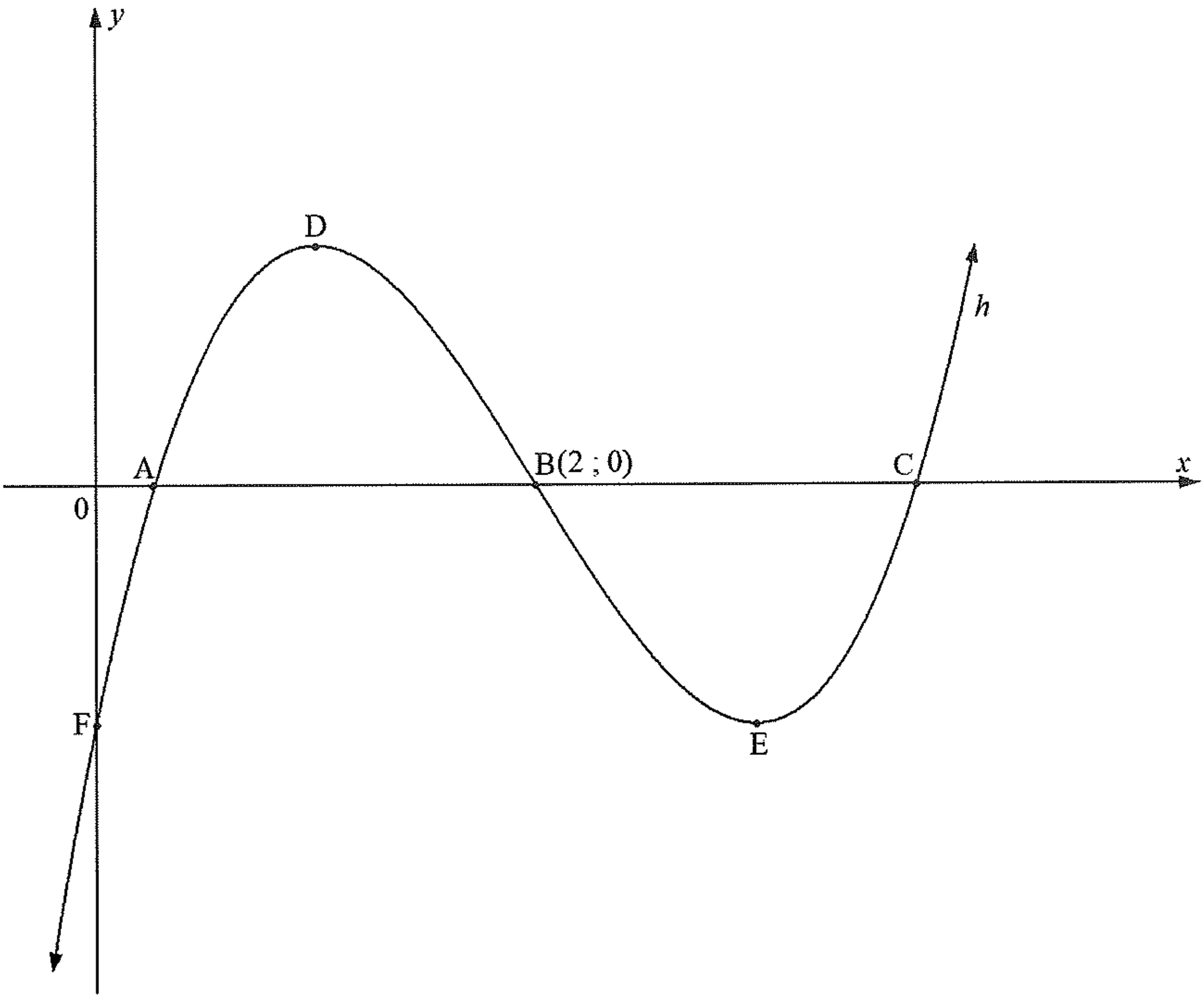
6.4.2 Hence, calculate $f^{\prime}(2)$ in terms of m (1)

6.4.3 Determine the numerical value of m if the gradient of the tangent to f at $x=2$ is equal to 39. (2)

[20]

QUESTION 7

The graph below shows a cubic function defined by $h(x) = x^3 + px^2 + 9x - 2$ which cuts the x-axis at A, B $(2 ; 0)$ and C. The graph of h cuts the y-axis at F and has turning points at D and E.



7.1	Write down the coordinates of F, the y-intercept of h.	(2)
7.2	Show that $p = -6$	(2)
7.3	Determine the length of BC. Leave the answer in surd form.	(5)
7.4	Hence, determine the coordinates of D and E.	(5)
7.5	Hence, write down the values of x for which $h(x) \times h'(x) > 0$ for $x > 2$	(3)
		[17]

QUESTION 8

The movement of a ball thrown from a ball-throwing machine forms a parabolic path given by the equation $h(t) = -t^2 + 6t + 1,62$ where h is the height (in metres) of the ball above the ground and t is the time in seconds.

8.1 Determine the initial height of the ball above the ground. (1)

8.2 Determine $h'(t)$. (1)

8.3 Hence, calculate the maximum height the ball reaches. (4)

8.4 Determine the height of the ball above the ground when it reaches a velocity (rate of change) of 3 m/s. (3)

[9]

QUESTION 9

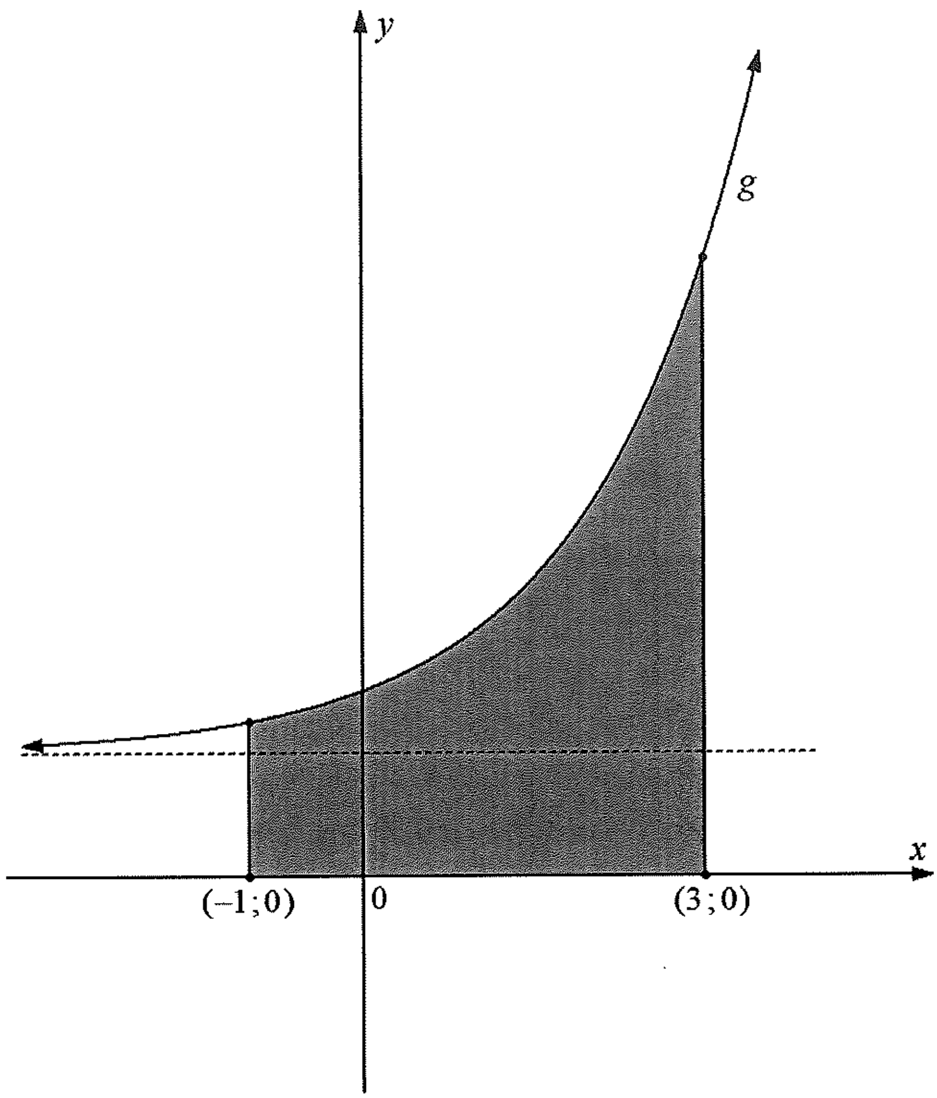
9.1 Determine the following integrals:

9.1.1 $\int x^4 \, dx$ (2)

9.1.2 $\int \left(2\pi + \frac{4}{x}\right) dx$ (2)

9.1.3 $\int \left(2x^{-3}\right)^3 \, dx$ (2)

9.2 The sketch below shows function g defined by $g(x) = 2^x + 2$ and the shaded area bounded by the curve of g and the x-axis between $x = -1$ and $x = 3$



Determine the area of the shaded region. Show ALL working details. (6)

[12]

TOTAL: 150

INFORMATION SHEET: TECHNICAL MATHEMATICS

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = -\frac{b}{2a}$ $y = \frac{4ac - b^2}{4a}$

$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0, a \neq 1 \text{ and } b > 0$

$A = P(1 + ni)$ $A = P(1 - ni)$ $A = P(1 + i)^n$ $A = P(1 - i)^n$

$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$ $\int kx^n \, dx = k \cdot \frac{x^{n+1}}{n+1} + C, \quad n \neq -1, \quad k \neq 0$

$\int \frac{1}{x} dx = \ln x + C, \quad x > 0$ $\int \frac{k}{x} dx = k \cdot \ln x + C, \quad x > 0, \quad k \neq 0$

$\int a^x \, dx = \frac{a^x}{\ln a} + C, \quad a > 0$ $\int k a^{nx} \, dx = k \cdot \frac{a^{nx}}{n \ln a} + C, \quad a > 0, \quad k \neq 0$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_2 + x_1}{2} ; \frac{y_2 + y_1}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\tan \theta = m$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc \cdot \cos A$

area of Δ ABC $= \frac{1}{2} ab \cdot \sin C$

$\sin^2 \theta + \cos^2 \theta = 1$ $1 + \tan^2 \theta = \sec^2 \theta$ $1 + \cot^2 \theta = \text{cosec}^2 \theta$

$\pi\, rad = 180°$

Angular velocity $= \omega = 2\pi n$ where n = rotation frequency

Angular velocity $= \omega = 360°\, n$ where n = rotation frequency

Circumferential velocity $= v = \pi D n$ where D = diameter and n = rotation frequency

Circumferential velocity $= v = \omega r$ where ω = angular velocity and r = radius

Arc length $= s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \dfrac{rs}{2}$ where r = radius, s = arc length

Area of a sector $= \dfrac{r^2\theta}{2}$ where r = radius and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a(m_1 + m_2 + m_3 + ... + m_n)$ where a = width of the equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$

$O_n = n^{th}$ ordinate and n = number of ordinates

OR

$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + ... + o_{n-1}\right)$ where a = width of the equal parts, $o_n = n^{th}$ ordinate and n = number of ordinates